

# Problem solving in mathematics

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The effective solving of mathematical problems entails several aspects, including knowing how to think and reason in mathematical terms, how to solve mathematical problems, how to learn mathematics and how to deal with maths anxiety. In this article I will focus on these and other aspects.

## 1 Constantly communicate with yourself while studying

- Form a complete picture of the work you are doing. Look at the titles of chapters, subsections and paragraphs. Study the graphs, diagrams and tables.
- Read, think and do:
  - Theory: Carefully read and analyse symbols. Learn facts until you know them by heart. Remember to revise all work within 24 hours. Use coloured pens to highlight important principles.
  - Problems: **Do** examples (NEVER just look at or read them!), talk about what you are doing and give reasons for everything you do. Devise your own examples, but use different values.

Divide your page in half and explain your reasons for everything you do as follows:

1.  $2x + 6 = 12$   
 $2x = 12 - 6$  Subtract 6 on both sides.
2. Solve a variety of problems. See in how many ways you can solve the same problem.
3. Do mixed examples and try to identify and name the different types of problems.

- Summary or framework. Now draw a framework with a mind map.
  1. Use a single surface to obtain the full picture.
  2. Keep it as brief as possible.
  3. Use core words, pictures, colours, block letters (print hand) and diagrams.
- Look at the following example of a possible framework:
  1. State the problem.
  2. Now describe the characteristics that enable you to recognise the problem.
  3. State what factual knowledge you will need to solve the problem.
  4. Now explain the method you will follow to solve it.
  5. Provide the worked out (model) solution.
  6. Answer a variety of questions obtained from old test and exam papers, class notes, worksheets and textbooks.
  7. Revision. Please keep the forget graph in mind! Learn and memorise all the definitions that you cannot yet repeat without referring to the textbook or notes.
  8. Develop a balance between solving examples and learning theory. Never neglect one of these tasks at the expense of the other.

*To summarise: Master mathematics actively. In mathematics you are advised to **actively** consider problems and examples before writing tests and exams, to work out examples repeatedly, to study the theory **thoroughly** until you really understand it and to **never** think that memorising facts can replace the **active** solving of problems.*

**Don't fool yourself:** It would be impossible to figure out every formula, principle, theorem or rule by referring to basic principles every time you find yourself in a test or exam situation. Certain aspect MUST be memorised. Think about your medical doctor – you listen when she explains to you where the origin of the pain is, what that specific part of your anatomy is called and which medication she prescribes.

### Replacing complex problems with simpler problems of the same kind

Try, as far as possible, to replace more difficult problems with simpler examples of the same kind. Once you have succeeded in solving simpler versions of problems, you will find the more difficult problems easier to solve. For example, replace the problem 'How much more than b is a?' with 'How much more than 9 is 5?' Or replace 'If Peter was m years old p years ago, how old is he now?' with 'If Peter was 13 years old 5 years ago, how old is he now?'

### Compile a system of mathematics files

You should strongly consider keeping the following system of files for mathematics:

- a file containing examples of principles, rules, theorems and evidence of what you should know by heart
- a file containing mathematics symbols
- a file containing mathematics terms
- a file containing model examples
- a file for writing notes on your personal mistakes
- a file in which all the aspects of fractions are clearly explained

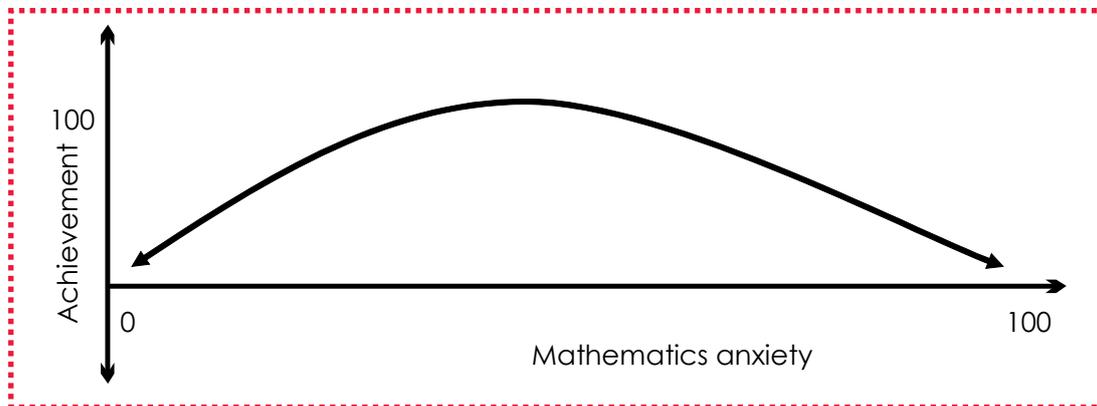
## 2 Dealing with maths anxiety

Maths anxiety has various causes. A learner experiencing maths anxiety will often say: 'Maths and I have never been friends,' or 'I've never been able to do maths.' Others will try to hide their anxiety by pretending that they do not care about the subject, or might try to look bored and disinterested. A relaxed, motivating and supportive atmosphere is particularly important in the mathematics class. Learners who are afraid of the teacher or lecturer will probably be afraid to ask questions if they do not understand the work.

Real-life mathematics can contribute significantly towards preventing maths anxiety. Consider the following: Rather than try to explain to a primary school learner that  $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$ , you should say, 'I jogged for half an hour and then walked for another quarter of an hour. For what fraction of an hour did I exercise?'

An important first step is to understand why children fear maths and do not enjoy it, and to take things from there. Mathematics is a very abstract subject. At school learners often do not get enough experience in doing 'real-life' maths – after all, you never meet an  $x$  or a  $y$  while walking along a street! Inadequate practising of solving maths problems and consequent uncertainty increase the likelihood of learners developing a fear of the subject. An understanding of the problem is nevertheless of major importance: You should continuously question the meaning of any maths problem to determine whether it fits into the larger maths picture.

Research conducted by me and some of my colleagues has revealed the following: A healthy degree of anxiety, especially anxiety that is linked with the ability to remain focused, shows that learners take their work seriously and is a reliable predictor of success. Therefore it would be worth your while to take note of the following graph:



Graph illustrating degree of mathematics anxiety in relation to achievement

A fear reaction that manifests physiologically (excessive sweating, fidgeting, tapping with a pen, blinking, a hollow feeling/butterflies in the stomach, tightness in the throat) is accompanied by negative thoughts ('I'm going to fail'; 'The paper is going to be difficult'; 'I don't know the '). **How does one cope with excessive fear of tests and examinations?**

- **Learn relaxation techniques.** Ask a teacher to help you or read a good book on the topic.
- **View the test or examination situation from another perspective.** Try to develop an understanding of the actual purpose of an exam, which is to help you to organise your knowledge and to identify your strengths and weaknesses. Studying would, after all, have no purpose if you were never required to give an account of what you have learnt. Last but not least: If doctors, anaesthetists and pilots, for example, were never required to prove that they possessed the qualifications and skills required for their occupations you would undoubtedly feel very unsafe if you had to undergo an operation or board a flight!
- **Ask someone to help** you decide on a realistic goal (e.g. should you aim for 56% or 86%?).
- **If your excessive fear of exams and tests is caused by emotional, social or other factors,** I implore you to please discuss the matter with your parents, your teacher or any other person whom you really trust.
- **If you become very anxious while preparing for an exam,** you should write about your problem on a piece of paper and stick it on to a surface where you cannot miss seeing it. Then tell yourself: I WILL NOT forget about this. I have to study now, but WILL attend to this once the exam is over. We are all aware of how much more daunting a problem seems late at night or shortly before an exam (when we unconsciously convince ourselves that there is nothing we can do about the situation). BE POSITIVE!
- **Not all stress is detrimental.** 'Friendly stress' and paralysing fear are totally different things. It is good to feel slightly nervous when studying or preparing for a test or exam as the adrenalin that your body produces has a beneficial effect and spurs you on to do your best. However, when your fear of exams and tests has a paralysing effect, prevents you from thinking clearly and causes you to keep 'striking a blank', it is advisable to seek the help of a professional.

### 3 Create mind maps or pictures in your mind

A mind map is a creative and innovative method for presenting related ideas. How does one draw a mind map?

- Decide what you want to present.
- Write or type the idea in the centre of a page or chart and draw a circle or a quadrangle around it.

Add a number of branches so that you can connect every related idea with the central idea.

- Now write key words and other details on the branches.
- Add symbols and picture to prompt your memory.

### 4 Use your senses when learning

Every learner has his/her own style of learning and usually uses **one** particular sense for presenting information. Approximately 40% of the population prefer to hear, talk, listen, sing or whisper (auditive learning), another 40% prefer to see (to look at charts, graphs, maps or pictures), while approximately 20% learn kinaesthetically, in other words, they like to write, walk or move around while learning. A small percentage of learners even prefer to use the senses of smell and taste for learning! Try to use as many of your senses as possible when studying. Read the study material aloud, talk to yourself (encourage yourself, use words with positive connotations), use colouring pens, smell the paper and books (!!), write out definitions, rules, theorems, facts, long questions and mind maps, and recite rules and principles while moving about. If you want to read through the work repeatedly, read it out loudly. Therefore: *Do*, read, write, talk, discuss, smell, taste and handle your work – just reading through it simply does not work. If possible, use all your senses – and never forget the importance of **common sense**. (You know very well that hard work is a prerequisite for excellence, and that too many late nights, cigarettes and alcohol abuse can negatively affect your marks. You do not need to be taught these basic principles by anyone!!)

### 5 How long should each study session be?

**To start with:** Divide your assignments into many smaller assignments. Study in short sessions and reward yourself after each session. Take time to go for a walk, listen to your favourite music or write messages to friends on *WhatsApp*. Do whatever you enjoy doing!

**Effective learning cannot take place unless you stop frequently to rest.** Do follow-up by taking another look at what you have already learnt as often as possible in order to consolidate your knowledge of the learning content. If you try to concentrate for too long, your concentration will fade and could even collapse completely. Optimal concentration can be ensured by taking a short break after every study session of 30–40 minutes and a longer break after every second session.

In most cases learners have to study the given work more than once to learn effectively. If you study the learning content twice, or even three or more times, you will remember it much better than you would after studying it thoroughly only once. Remember: Go through your work as soon as possible after every lesson! Sometimes it is necessary to learn work as many as 10 times before you really know it. A person's acquisition of knowledge often reaches a ceiling (after a number of attempts, there is no real increase in the knowledge you have acquired). It is exactly then that you have to decide to persevere and continue repeating the learning process.

The principle of 're-learning' therefore has to be applied consistently (in other words, study the work over and over even after you have developed a thorough understanding of the learning content). You will remember much more through re-learning. Do revision soon, and repeat it often.

## 6 Achieve success in your examination

Remember: Your success will ultimately be measured by how well you do in the examinations!

### • Preparing for an examination

- Mathematics has to be learnt or mastered *actively*. Work on your own, solve as many types of problems as possible.
- Try to get approximately eight hours' sleep the night before your exam.
- Try to complete your preparation long before the exam date as you ought to have some time to relax before that time.
- Use your year planner to plan the time set aside to prepare for tests and exams. Remember: No subject can be mastered in the home stretch – this is a race that requires sustained effort from beginning to end.
- Plan your revision. Work through old test and exam papers and ask your teacher to mark your work.
- Test your knowledge orally and in writing.
- During your preparation you should focus on those parts of the work with which you usually experience problems.
- Prepare yourself mentally and psychologically for tests and exams. BELIEVE in yourself and your ability to excel.
- Read through the list of contents in your textbook and test yourself by checking whether you understand and can express everything in words.
- Enjoy a light meal before your test or exam, but avoid drinking too much coffee and tea. Make sure that you always get enough exercise.
- Get everything you will need ready well in advance – pen and pencil, eraser, ruler, protractor, set squares, your watch, a calculator (remember the fresh batteries) and headache tablets.
- Make sure that you are wearing comfortable clothes.
- Visit the cloakroom just before the exam starts.
- Do not scan the questions before you start writing.
- Imagine that you are a batsman facing a ball, an athlete ready to start a race – prepare yourself mentally.
- Before any test or exam, find out *exactly* which sections of the work the questions will cover, what you can expect and how long the paper will be.

### • Writing the examination

A few principles that should be borne in mind in this regard are:

- Do not chat to friends just before entering the examination venue – rather think about a pleasant experience. Friends could ask questions or make remarks that may give rise to uncertainty.
- Study each question thoroughly and make sure that you understand exactly what is required before answering it.
- Keep an eye on the clock, but view it as a mechanical aid and not as a terrifying monster.
- If you feel uncertain about one or two questions, relax – your friends will probably also struggle with them. Always start with the sums that you can do with confidence. Do not look at your friends – their facial expressions could confuse or upset you.
- Never spend too much time on questions that you cannot answer immediately.
- Do not panic if you strike a blank, in other words, if you suddenly get the feeling that you know absolutely nothing. Relax, look at something green, think about something pleasant that happened in the past and the mental block will automatically disappear after a few minutes.
- Remain *active* all the time – do, think, draw mind maps and sketches where necessary.
- Never put a line through your answers or erase them before you have answered the question or solved the problem or sum in another way.
- Always show a few steps when using a calculator to do sums.
- Read the instructions given on your paper *with care*.
- If you finish before the time is up: Check your answers thoroughly. Recalculate the answers about which you feel uncertain. Do this on scrap paper – mistakes often become evident only when the calculation is repeated.

- Leave enough space for questions that you could not answer initially. In many respects your brain is like a complex computer and processes the information (the difficult question) in its own way and in its own time. When you later return to that question it is very likely that you will be able to answer it.
- In mathematics you should *always* determine whether it is perhaps necessary (or possible) to use the solution to one part of the problem when solving the next part.
- Make sure that your work is legible and understandable. Sufficient spacing is important.
- Do not reproach yourself for not being able to answer some of the questions – rather feel good about every question you could answer. Encourage yourself.
- Relax once you have finished writing the exam – be kind to yourself. Reward yourself by doing something you enjoy.

• **General hints**

1. **Study venue**

The temperature in the room should be comfortable and there should be enough light and fresh air. Ensure that your body does not cast a shadow on your book when you are studying or writing. Lying down while studying is never a good idea! Do not start studying in new surroundings shortly before an exam. Avoid studying in front of a window where people/objects outside could continually cause distractions.

2. **Is one study method better than another?**

Some learners prefer to learn directly from their textbooks (they underline important points) while others make telegram-style summaries (using the kind of language that is used in ‘texting’) of the work. Another group prefers to use mind maps, diagrams, sketches and frameworks. One specific method is not necessarily good for all learners – each learner adapts study methods to suit his/her personal needs, situation and style.

## 7 Metalearning, metareasoning, cognition and metacognition – the bases of effective learning and excellence in mathematics

Metalearning and metareasoning refer to, among other things, new measures that can be taken to ensure that you actually learn and think effectively. Some people regard these concepts as being among the most important aspects promoting excellence. Cognition refers to what people do when they are thinking or learning, while metacognition refers to the measures or methods (metalearning and metareasoning) that they apply to ensure effective thinking and reasoning.

**Metacognition entails the following steps:**

1. **Effective planning**

Start with a plan of action. The *how* of your activity is important. Ask yourself questions such as the following:

- How do I prepare myself to ensure that I will pass a test or exam?
- How can I do this difficult sum?

Then take responsibility for monitoring your activities yourself.

2. **Self-monitoring**

This means that from time to time you will have to ‘step outside yourself’, stand aside and look at your effort objectively. The above-mentioned activities could, for instance, be monitored as follows:

- Do I learn actively, or do I simply sit behind my books thinking about other things while the time passes?
- Do I know exactly what I was asked and what is required of me? Have I thought of a way in which to do the sum? Have I already linked the new information with the information previously mastered?

However, you should also be able to evaluate your activities.



Learn how on the next page

### 3. Self-evaluation

Self-evaluation is the next step after self-monitoring. Ask yourself questions such as the following:

- Am I able to ask myself questions (in writing and orally) and answer them?
- Am I able to look at my own efforts objectively? What would my opinion have been if they were someone else's effort?
- Am I able to look back and decide whether my solution offers a possible answer to the original question, or whether it perhaps makes no sense? Let us look at the following example: If 12 men are able to complete a task in 4 days, how many men will be required to complete the same task in 2 days? If my answer is  $12\frac{1}{2}$  men, I will know that it makes no sense and that I will have to try again!

If you discover any mistakes or gaps, you have to be able to correct them.

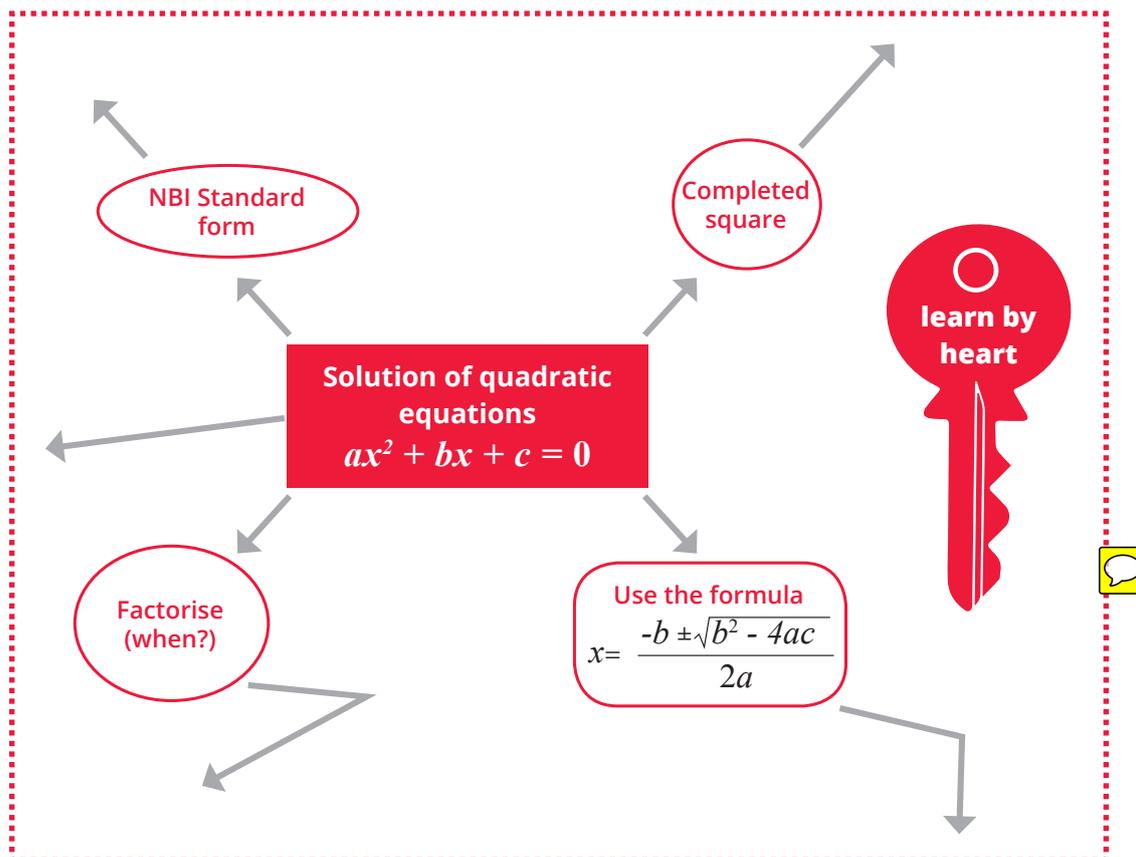
### 4. Correction of mistakes

If learners discover that they have not been successful enough, they should be able to apply remedial measures. This means that they will have to modify or perhaps drastically change their initial strategies or plans. The following are some examples of what may be required:

- Studying while watching and listening to the TV simply does not work – I learn more effectively in a quiet space.
- Before I continue with my work, I have to discuss my emotional problems with someone.
- I am simply not studying enough, so I will have to limit the time I spend relaxing.
- It is important that, in the future, I should first make sure that I understand the 'old' work as well as work I have to learn before tackling difficult problems.

The process of correcting mistakes will differ from one case to the next and there will not always be a clear, watertight division between the different steps. That is the nature of metacognition. The idea is to ensure that learners realise that they have to be able to evaluate the way in which they do things. If necessary, they should be able to change direction or make changes (sometimes guided by an adult).

Example of mind map:



## 8 Focus on the 'emotional' side of achievement in mathematics

Nowadays it is generally accepted that focusing only on the so-called cognitive side of achievement in mathematics is short-sighted. In other words, if a learner achieves 33% for maths, it is essential to determine the reason for his/her poor performance. Discussion is therefore of crucial importance in maths classes. Learners could, for example, be asked to complete a short questionnaire like the one below to determine whether their study methods are appropriate. Problem areas should of course be addressed as soon and as professionally as possible.

### Study habits questionnaire for learners in mathematics classes

Circle either 'Yes' or 'No' in the shaded columns to indicate your answer to each of the questions.



I	Yes	No
1. do my homework every day.	Yes	No
2. listen attentively to my maths teacher's instructions and explanations during lessons.	Yes	No
3. mark my work every day and make sure that I correct all the wrong answers.	Yes	No
4. participate actively in discussions in the maths class.	Yes	No
5. do follow-up after tests/exams to see what mistakes I made.	Yes	No
6. never copy other learners' solutions to maths problems.	Yes	No
7. am thoroughly familiar with important rules, theorems and formulae that are applicable to mathematics.	Yes	No
8. work too fast.	Yes	No
9. work too slowly.	Yes	No
10. explain the reasons for every step used to arrive at the answer.	Yes	No
11. check my answers to maths problems.	Yes	No
12. estimate my answers to maths problems where possible.	Yes	No
13. am good at doing calculations involving fractions.	Yes	No
14. try to rewrite difficult sums in a simpler form with which I am more familiar and that I understand better.	Yes	No
15. work systematically.	Yes	No
16. do <i>more</i> than is expected of me by my teacher (do extra homework in addition to the work given to me by my teacher).	Yes	No
17. draw a red frame around <i>all</i> the important principles.	Yes	No
18. do the sums in advance (to ensure that I can follow the work easily when we get to it in class).	Yes	No
19. use coloured pens to highlight important principles.	Yes	No
20. try to work through old test and exam papers.	Yes	No
21. <i>do</i> examples actively.	Yes	No
22. plan my time to allow for enough time for studying before tests and examinations.	Yes	No
23. make sure that I do not use my calculator wrongly.	Yes	No
24. am careless.	Yes	No
25. make elementary mistakes when calculating.	Yes	No
26. allocate wrong number values to mathematical symbols.	Yes	No

Now award yourself 1 mark for every 'Yes' and 0 for every 'No' in questions 1 to 24 and 0 for every 'Yes' and 1 for every 'No' in questions 24 to 26 and decide whether the way you study for mathematics is satisfactory or not.

(Any score below 26 is unacceptable and every 'No' answer should be followed up!)

## 9 In conclusion

Gladwell (2008) and others often refer to the 10 000-hour principle that should be applied if you wish to achieve success: It takes a person 10 000 hours to become a master in any field (think of people like Einstein, Roger Federer, Cristiano Ronaldo, Bryan Habana ...), including mathematics. In other words, unless you are prepared to work hard at mastering this subject, your chances of achieving success are very slim. To help you to understand what I mean, I recommend that you read the story of the *Beatles in Hamburg* on Google.

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**Good luck!**

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